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Do Heterogeneous Expectations Constitute a Challenge for Policy Interaction?

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Do Heterogeneous Expectations Constitute a Challenge for Policy Interaction?☆

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Abstract

Yes, indeed; at least when it comes to fiscal and monetary policy interaction. We examine a Neo-Classical economy, where agents have either rational or adaptive expectations. We demonstrate that the monetarist solution can be unique and stationary under a passive fiscal/active monetary policy regime, because active monetary policy incorporates expectational heterogeneity. In contrast, under an active fiscal/passive monetary policy regime, the fiscalist solution is prone to explosive dynamics due to empirically plausible expectational heterogeneity. However, conditional on stationarity, both regimes can yield promising business cycle dynamics, which are absent in the homogeneous expectations benchmark.

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1. INTRODUCTION

Modeling expectations in modern macroeconomics is dominated by the paradigm of homogeneous expectations. Even when a continuum of agents is assumed, routinely subjective expectations coincide with the aggregate average expectations as symmetry among agents is imposed.

The prevalence of homogeneous expectations reaches far beyond the dominating rational expectations hypothesis (REH) into the literature on bounded rationality. One example is the standard adaptive learning approach, see e.g. Evans and Honkapohja (2001).

However, recent empirical and experimental research provides compelling evidence undermining the homogeneous expectations hypothesis. Evidence in favour of the heterogeneous expectations hypothesis based on survey data can be found in Branch (2004) or Bovi (2013). Hommes (2011) and Assenza et al. (2011) document the pervasiveness of heterogeneous expectations in laboratory experiments. Hommes (2011) also provides an elaborate review of the vast evidence in favour of the heterogeneous expectations hypothesis.

These findings have triggered a notable number of studies tackling the issue of how expectational heterogeneity may affect aggregate economic dynamics. Examples are the seminal work of Brock and Hommes (1997) on dynamic predictor selection, or the contributions of Branch and Evans (2006) and Branch and McGough (2009) in monetary economics amongst others.

Nonetheless, the issue of fiscal and monetary policy interaction, so far, has only been examined under the homogeneous expectations hypothesis. This is somewhat surprising given the findings that not only fiscal and monetary policy interaction, but also the expectational set-up can have important consequences for the determination of the price level. Prominent examples for analyses under homogeneous expectations are Leeper (1991) and Evans and Honkapohja (2007). The core question in this strand of the literature is whether or not fiscal variables affect the price level. In fact, depending on the policy regime, typically two unique stationary rational expectations equilibria (REE) are possible.[1] One is routinely denoted the *fiscalist* solution and involves the price level depending on fiscal variables, whereas the other one is usually referred to the *monetarist* solution, in which the price level does not depend on fiscal variables.

Our primary contribution is to address the issue of whether fiscal variables can affect inflation. The key novelty of our paper is that we embed fiscal and monetary policy interaction à la Leeper (1991) into a heterogeneous expectations set-up à la Branch and McGough (2009). Agents either have *rational* (RE) or *adaptive expectations* (AE). One can interpret such a set-up as one of persistent heterogeneity. Evans and Honkapohja (2013) put forth the argument that this is a plausible assumption, even when agents may entertain various forecasting models.

Despite the fact that such a modeling approach partly neglects the plurality of predictors that the afore-mentioned evidence suggests, it is appealing for at least three reasons. First, a common feature of the evidence is the presence of a relatively large share of agents with AE among agents with other, more or less sophisticated predictors. Branch (2004) provides evidence for a share of agents with AE and its special case of naïve expectations around 47%. Second, this approach allows for analytical tractability, and third, the model nests the RE benchmark case. The latter facilitates a direct comparison to the related literature on fiscal and monetary policy interaction. As a matter of fact, comparability is also one

[1] A situation in which there exists a unique stationary REE is referred to local *determinacy*. Moreover, local *indeterminacy* denotes the existence of multiple stationary REE. Finally, if no stationary REE exists, the economy is said to feature local *divergence* or *explosiveness*.

of our motives to limit the analysis to a Neo-Classical economy as studied by Leeper (1991) or Evans and Honkapohja (2007).[2]

Assuming expectational heterogeneity in this particular way introduces a new state variable to the economy, namely lagged inflation. This eventually changes the dynamic properties of the economy and the resulting policy implications. Actually, we show that, when focusing on the determinate cases, different restrictions on RE solutions can emerge. One of them involving inflation depending on fiscal variables, i.e. the fiscalist solution, whereas others do not, i.e. monetarist solutions.

Subsequently we examine the full set of REE and find that four different types of stationary solutions are possible. We relate the four types of solutions to different policy regimes and show under which conditions the shares of agents with RE and AE have a crucial role in determining economic outcomes. Our main result is that whether or not the fiscalist solution is stationary, turns out to depend crucially on the share of agents with RE. In contrast, non-explosiveness of the monetarist solution appears to be less vulnerable to the presence of heterogeneous expectations. This can be explained by the extent to which a policy incorporates heterogeneous private sector expectations.

Finally, we ask how the economy responds to a transitory, contractionary monetary, or a negative fiscal policy shock under the different policy regimes in which determinacy prevails. The impulse responses to both shocks appear to have striking characteristics. First, in contrast to the homogeneous RE benchmark case, the impulse responses of inflation under heterogeneous expectations exhibit significant persistence. This feature is absent from the benchmark RE

[2]Clearly our analysis can be extended to a New-Keynesian economy. We pursue this goal in a related paper.

Neo-Classical model, when monetary policy shocks occur. The lack of persistence was one of the main motivations for the introduction of nominal rigidities over the last decades, and is not an uncontroversial issue in the profession. In our model nominal rigidities to generate persistence are obsolete, as long as there is reasonable heterogeneity. Second, in case of the monetary policy shock under heterogeneous expectations, the impact effects can have opposite sign, as expectational heterogeneity not only introduces lagged inflation to the inflation dynamics, but also amplifies the influence of fiscal variables. Finally, convergence occurs in dampening oscillations. Such interesting business cycles dynamics are uncommon to homogeneous expectations models, but an intrinsic feature of our model.

The remainder of the paper is organized as follows. In Section 2 we present a simple Neo-Classical economy under heterogeneous expectations and derive the aggregate equilibrium conditions from individual behaviour. Section 3 analyzes the dynamic properties of the model, condenses our main results into two propositions and ends with an impulse response analysis. In Section 4 we discuss the policy implications of our results, before a conclusion summarizes in Section 5.

2. THE MODEL

We develop our analysis in a heterogeneous expectations version of the model outlined in Evans and Honkapohja (2007). We consider infinitely many households and each individual household's utility depends on real consumption, c_s, and beginning of period real money balances, $\pi_s^{-1} m_{s-1}$. The household's maximization problem is given by

$$\max_{c_s, m_s, b_{s-1}, m_{s-1}} \quad E_t^\gamma \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{c_s^{(1-\sigma_1)}}{(1-\sigma_1)} + \mathcal{A} \frac{\left(\frac{m_{s-1}}{\pi_s}\right)^{(1-\sigma_2)}}{(1-\sigma_2)} \right] \right\} \tag{1}$$

$$\text{s.t.} \quad c_s + m_s + b_s + \tau_s = y + \frac{m_{s-1}}{\pi_s} + R_{s-1} \frac{b_{s-1}}{\pi_s}, \tag{2}$$

where (2) is the household's budget constraint. Moreover, $0 < \beta < 1$ is the discount factor, $0 < \sigma_1, \sigma_2 < 1$ are the elasticities of substitution, and $\mathcal{A}$ is a relative weight on real balances. $y > 0$ is a constant endowment and b_s are end-of-period holdings of real bonds respectively. Next, τ_s are real lump-sum taxes and R_{s-1} is the gross nominal interest rate paid at the beginning of s. Note that this rate is pre-determined. Finally, the government is assumed to purchase and waste constant $g \geq 0$ in each period.

The subjective expectations operator of a household that is of type γ is denoted $E_t^\gamma\{\cdot\}$. We assume that all households are perfectly identical apart from the way they form expectations. In this regard, a household is considered to be of one of the two types $\gamma \in \{1, 2\}$. Following the heterogeneous expectations set-up of Branch and McGough (2009), for any variable q_t we have

$$E_t^1 q_{t+1} = E_t q_{t+1}, \tag{3}$$

$$E_t^2 q_{t+1} = \iota E_t^2 q_t = \iota^2 q_{t-1}, \text{ and} \tag{4}$$

$$\widehat{E}_t q_{t+1} = \chi E_t q_{t+1} + (1-\chi)\iota^2 q_{t-1}. \tag{5}$$

Here χ is the share of agents of type $\gamma = 1$ forming RE as in (3). Agents of type $\gamma = 2$ form AE for unobserved and next period variables, and ι is the coefficient that these agents use to forecast variables based on the most recent observation according to (4). We restrict the coefficient to $\iota > 0$ and consider the cases

$\chi \in (0, 1]$.[3]

In Appendix B we show that optimal behaviour of households and market clearing conditions yield the *Fisher relation* and a money market clearing condition in period t given by

$$R_t^{-1} = \beta E_t^{\gamma}\{\pi_{t+1}^{-1}\}, \quad \text{and} \tag{6}$$

$$\mathcal{A}\beta m_t^{-\sigma_2} E_t^{\gamma}\{\pi_{t+1}^{\sigma_2 - 1}\} = (y-g)^{-\sigma_1}(1 - \beta E_t^{\gamma}\{\pi_{t+1}^{-1}\}) \tag{7}$$

respectively. Notice that it is also necessary to impose the transversality conditions

$$\lim_{t\to\infty} \beta^t m_{t+1} = 0, \quad \text{and} \quad \lim_{t\to\infty} \beta^t b_{t+1} = 0. \tag{8}$$

Next, the government budget constraint in real terms is given by

$$b_t + m_t + \tau_t = g + \frac{m_{t-1}}{\pi_t} + R_{t-1}\frac{b_{t-1}}{\pi_t}. \tag{9}$$

It basically states that government spending and interest payments on debt outstanding can be funded by issuing new debt, seigniorage, and taxes.

Following Leeper (1991), we assume two public authorities that interact with each other. First, there is a fiscal authority, which follows the tax rule

$$\tau_t = \gamma_0 + \gamma b_{t-1} + \psi_t. \tag{10}$$

The rule implies that the authority responds to previous period real debt and exogenous fiscal policy shocks, ψ_t. Second, there is a central bank conducting

[3] See Appendix A for more details about the assumptions on the subjective expectations operator.

monetary policy according to the interest rate rule

$$R_t = \alpha_0 + \alpha\pi_t + \theta_t. \tag{11}$$

Thus, this rule relates the central bank's policy instrument to its mandate of controlling inflation. Moreover, monetary policy shocks, θ_t, might occur. Here θ_t and ψ_t are assumed to be exogenous *iid* mean zero random shocks. The feedback of policy to the targeted variable is governed by the coefficients γ and α. As one can see later on, these coefficients determine qualitatively different types of fiscal and monetary policies.

According to Leeper (1991) and Evans and Honkapohja (2007), the fiscal authority has an *active fiscal policy* stance (AF), if $|\beta^{-1} - \gamma| > 1$. In contrast, fiscal policy is considered to be *passive* (PF), if $|\beta^{-1} - \gamma| < 1$. For the central bank, monetary policy is *active* (AM), if $|\alpha\beta| > 1$ and *passive* (PM) if $|\alpha\beta| < 1$.

The aforementioned authors explain that, for the empirically realistic case, $0 \leq \gamma < \beta^{-1}$, AF implies that under rule (10) the additional tax revenue generated from a small increase in the steady-state level of debt is lower than the increase in the related interest payments. For PF, the reverse is true. Moreover, $\alpha > 0$ implies a positive response of the real interest rate to an increase in steady-state inflation.

3. DYNAMICS UNDER POLICY INTERACTION

3.1. Determinacy Properties

A linearized version of the economy (6)-(7), including the policy block (9) to (11) as well as the expectational set-up (3) to (5), can be expressed by a two-dimensional system[4]

$$\tilde{\pi}_t = (\alpha\beta)^{-1}\chi E_t\tilde{\pi}_{t+1} + (\alpha\beta)^{-1}(1-\chi)\iota^2\tilde{\pi}_{t-1} - \alpha^{-1}\theta_t \tag{12}$$

$$\begin{aligned} 0 &= \tilde{b}_{t+1} + \varphi_1\chi E_t\tilde{\pi}_{t+1} + \varphi_1(1-\chi)\iota^2\tilde{\pi}_{t-1} + \varphi_2\tilde{\pi}_t \\ &\quad - (\beta^{-1}-\gamma)\tilde{b}_t + \psi_{t+1} + \varphi_3\theta_{t+1} + \varphi_4\theta_t, \end{aligned} \tag{13}$$

where, as in Evans and Honkapohja (2007),

$$\varphi_1 = [\tilde{C}\beta\alpha + m\pi^{-2} + Rb\pi^{-2}], \qquad \varphi_2 = [-\pi^{-1}\tilde{C}\beta\alpha - \pi^{-1}b\alpha],$$
$$\varphi_3 = \tilde{C}\beta, \qquad \varphi_4 = [-\pi^{-1}\tilde{C}\beta - \pi^{-1}b].$$

Following their example, we abstract from the special cases $\alpha = 0$, $\alpha\beta \neq 1$, $\gamma\beta \neq 1$, and $\beta^{-1} - \gamma \neq 1$ in what follows.

The system can be rearranged as

$$\begin{bmatrix} \tilde{\pi}_t \\ \tilde{b}_t \\ \tilde{\pi}_{t-1} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \tilde{\pi}_{t+1} \\ \tilde{b}_{t+1} \\ \tilde{\pi}_t \end{bmatrix} + \mathbf{F}_1\eta_{t+1} + \mathbf{F}_2\theta_{t+1} + \mathbf{F}_3\theta_t + \mathbf{F}_4\psi_{t+1}, \tag{14}$$

where $\mathbf{J}$ is the Jacobian of the system given by

$$\mathbf{J} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & (\beta^{-1}-\gamma)^{-1} & \frac{((\alpha\beta)\varphi_1+\varphi_2)}{(\beta^{-1}-\gamma)} \\ -\frac{\chi}{\Theta} & 0 & \frac{(\alpha\beta)}{\Theta} \end{pmatrix}.^{5} \tag{15}$$

Note that $\eta_{t+1} = \tilde{\pi}_{t+1} - E_t\tilde{\pi}_{t+1}$ is a *martingale difference sequence* as we assume

[4]Any variable $\tilde{q}_t$ represents the respective variable in log-deviations.

[5]Note that information regarding any matrice not reported herein is not relevant for the

$E_t \eta_{t+1} = 0$. We also define $\Theta \equiv (1-\chi)\iota^2$ for notational convenience. In Appendix C we show that $\lambda_1 \equiv (\beta^{-1}-\gamma)^{-1}$, $\lambda_2 \equiv \frac{(\alpha\beta)-\sqrt{(\alpha\beta)^2-4\Theta\chi}}{2\Theta}$, and $\lambda_3 \equiv \frac{(\alpha\beta)+\sqrt{(\alpha\beta)^2-4\Theta\chi}}{2\Theta}$ are the eigenvalues of $\mathbf{J}$.

The crucial difference between the economy in Evans and Honkapohja (2007) and the one herein is, that the latter involves the dynamics of one free and two predetermined variables in presence of heterogeneous expectations, i.e. $\chi < 1$. The additional state variable is lagged inflation. This has important consequences for the question of when a REE is determinate.

Technically speaking, determinacy requires that the number of eigenvalues inside (outside) the unit circle matches the number of free (predetermined) variables, which is one (two) in our case. If the number of eigenvalues inside the unit circle exceeds the number of free variables, then the economy exhibits divergence from the local steady-state. In contrast, if the number of eigenvalues inside the unit circle is smaller than the number of free variables, the economy is said to be locally indeterminate.

One of our main goals is to relate qualitatively different economic dynamics to certain policy regimes. Therefore, we have to refine the notion of AM and PM. First, note that it is obvious that $|\lambda_1| > 1$ if $|(\beta^{-1} - \gamma)| < 1$ is the case. This corresponds to PF and the reverse is true in case of AF. Second, we will denote $(\alpha\beta) < \chi + \Theta$ *passive monetary policy under heterogeneous expectations* (PMHE), which corresponds to PM for $\chi = 1$. Third, we denote $(\alpha\beta) > \chi + \Theta$ *active monetary policy under heterogeneous expectations* (AMHE), which corresponds to AM for $\chi = 1$.

For the moment, let us focus on the determinate cases. In Appendix C, we

analysis and omitted for clarity of exposition. Of course this information is available from the author upon request.

argue that linear restrictions of the type

$$\tilde{\pi}_t = K_1 \tilde{b}_t + K_2 \theta_t + K_3 \tilde{\pi}_{t-1} \tag{16}$$

emerge and yield a stationary solution. In particular we state that:

(i) In the case of AF/PMHE, $|\lambda_1| < 1$, and $|\lambda_2|, |\lambda_3| > 1$, the coefficients are given by

$$\begin{aligned}
K_1 &= \left[\frac{\sqrt{(\alpha\beta)^2 - 4\Theta\chi}(\beta^{-1} - \gamma)(\lambda_1 - \lambda_3)(\lambda_1 - \lambda_2)}{\chi[(\alpha\beta)\varphi_1 + \varphi_2](\lambda_3 - \lambda_2)} \right], \\
K_2 &= \left[\frac{\sqrt{(\alpha\beta)^2 - 4\Theta\chi}(\lambda_1 - \lambda_3)(\lambda_1 - \lambda_2)}{\chi(\lambda_3 - \lambda_2)} \right] \times \\
&\left[\frac{\beta}{[(\alpha\beta) - \lambda_1\Theta - (\beta^{-1} - \gamma)\chi]} - \frac{(\beta\varphi_1 + \varphi_4)}{[(\alpha\beta)\varphi_1 + \varphi_2]} \right], \quad \text{and} \\
K_3 &= \frac{\Theta}{\chi}\lambda_1;
\end{aligned}$$

(ii) In the case of PF/AMHE, $|\lambda_1| > 1$, $|\lambda_2| < 1$, and $|\lambda_3| > 1$, the coefficients are given by $K_1 = 0$, $K_2 = -\chi^{-1}\beta\lambda_2$, and $K_3 = \chi^{-1}\Theta\lambda_2$;

(iii) In the case of PF/PMHE, $|\lambda_1|, |\lambda_2| > 1$ and $|\lambda_3| < 1$, the coefficients are given by $K_1 = 0$, $K_2 = -\chi^{-1}\beta\lambda_3$, and $K_3 = \chi^{-1}\Theta\lambda_3$.

In the homogeneous RE version of this economy a PF/PM regime leads to indeterminacy and an AF/AM regime yields local divergence. For this reason, we now ask, to what extent these findings carry over to the heterogeneous expectations version.

In order to do so, we examine the whole set of REE. By defining $y_t \equiv [\tilde{\pi}_t, \tilde{b}_t, \tilde{\pi}_{t-1}]'$ and $v_t \equiv [\theta_t, \psi_t]'$, we can recast the economy (12)-(13) as

$$y_t = \mathbf{M} E_t y_{t+1} + \mathbf{N} y_{t-1} + \mathbf{P} v_t + \mathbf{R} v_{t-1}, \tag{17}$$

where

$$\mathbf{M}=\begin{pmatrix}(\alpha\beta)^{-1}\chi & 0 & 0\\ -\varphi_1(\alpha\beta)^{-1}\chi & 0 & 0\\ 0 & 0 & 0\end{pmatrix},\qquad \mathbf{N}=\begin{pmatrix}(\alpha\beta)^{-1}\Theta & 0 & 0\\ -\varphi_1(\alpha\beta)^{-1}\Theta-\varphi_2 & \beta^{-1}-\gamma & 0\\ 1 & 0 & 0\end{pmatrix},$$

$$\mathbf{P}=\begin{pmatrix}-\alpha^{-1} & 0\\ \varphi_1\alpha^{-1}-\varphi_3 & -1\\ 0 & 0\end{pmatrix},\quad \text{and}\quad \mathbf{R}=\begin{pmatrix}0 & 0\\ -\varphi_4 & 0\\ 0 & 0\end{pmatrix}. \tag{18}$$

We assume that REE follow

$$y_t=\mathbf{A}+\mathbf{B}y_{t-1}+\mathbf{C}v_t+\mathbf{D}v_{t-1}. \tag{19}$$

In consequence, the very same *undetermined coefficient* reasoning as in Evans and Honkapohja (2007, p.678) leads to the following proposition.

PROPOSITION 1. *One can verify that there exist four types of solutions:*

(I) One solution is characterized by satisfying restriction (i) *and matrix* $\mathbf{B}=(\kappa\chi)^{-1}\times$

$$\begin{pmatrix}-\beta\Theta\varphi_1-\left(\alpha\beta^2+(\beta\gamma-1)\chi\right)\varphi_2 & -\beta^{-1}\left[(\alpha(\beta\gamma-1)+\Theta)\beta^2+(\beta\gamma-1)^2\chi\right] & 0\\ \beta\left(\Theta\varphi_1^2+\alpha\beta\varphi_2\varphi_1+\chi\varphi_2^2\right) & \beta(\alpha(\beta\gamma-1)+\Theta)\varphi_1+(\beta\gamma-1)\chi\varphi_2 & 0\\ 1 & 0 & 0\end{pmatrix},$$

where $\kappa\equiv(\beta\gamma-1)\varphi_1-\beta\varphi_2$*.* $\mathbf{A}=0$*, and* $\mathbf{C}$ *as well as* $\mathbf{D}$ *are also uniquely determined. In this case, the eigenvalues of matrix* $\mathbf{B}$ *are* $\{0,\chi^{-1}\Theta\lambda_2,\chi^{-1}\Theta\lambda_3\}$*. We denote this the fiscalist solution under heterogeneous expectations. In case of* $\chi=1$ *this solution corresponds to the traditional fiscalist solution.*

(II) A second solution satisfies restriction (ii) *with matrices* $\mathbf{B}=$

$$\begin{pmatrix} \chi^{-1}\Theta\lambda_3 & 0 & 0 \\ -\chi^{-1}\Theta\lambda_3\varphi_1 - \varphi_2 & \lambda_1^{-1} & 0 \\ 1 & 0 & 0 \end{pmatrix}$$, *and* $\mathbf{A} = 0$. *Moreover,* $\mathbf{C}$ *and* $\mathbf{D}$ *are uniquely determined. The eigenvalues of matrix* $\mathbf{B}$ *are* $\{0, \lambda_1^{-1}, \chi^{-1}\ \Theta\lambda_3\}$. *This can be denoted the monetarist solution under heterogeneous expectations. For* $\chi = 1$ *this solution is the traditional monetarist solution.*

(III) A third solution, satisfying restriction (iii), is possible and is characterized by matrices $\mathbf{B} = \begin{pmatrix} \chi^{-1}\Theta\lambda_2 & 0 & 0 \\ -\chi^{-1}\Theta\lambda_2\varphi_1 - \varphi_2 & \lambda_1^{-1} & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $\mathbf{A} = 0$, *and* $\mathbf{C}$ *and* $\mathbf{D}$ *uniquely determined. The eigenvalues of matrix* $\mathbf{B}$ *are* $\{0, \lambda_1^{-1}, \chi^{-1}\Theta\lambda_2\}$. *Again, this solution states nothing but the monetarist solution for* $\chi = 1$.

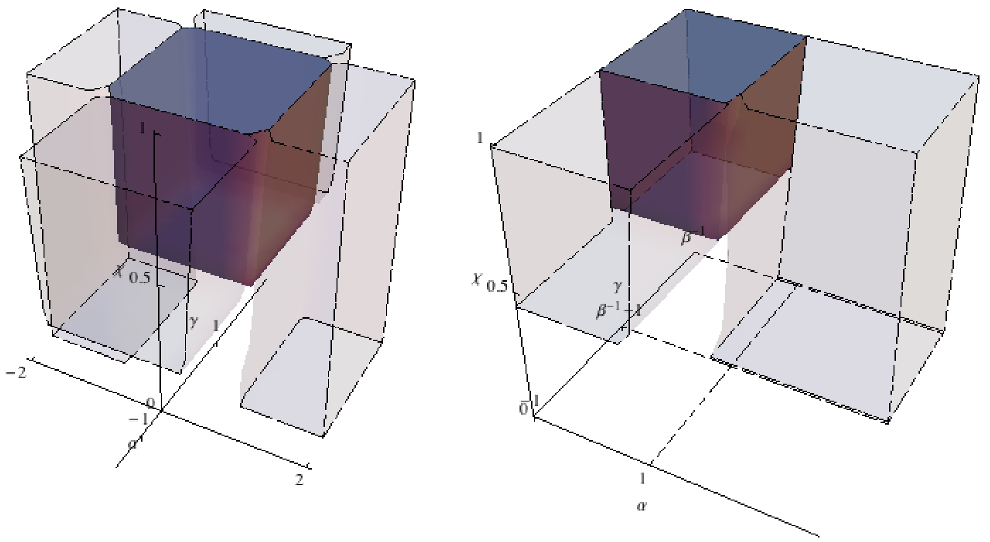
(IV) Finally, there is a continuum of non-fundamental solutions characterized by matrices $\mathbf{B} = \begin{pmatrix} \chi^{-1}(\alpha\beta) & 0 & -\chi^{-1}\Theta \\ -\chi^{-1}(\alpha\beta)\varphi_1 - \varphi_2 & \chi^{-1}\Theta\varphi_1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, *and* $\mathbf{A} = 0$. *However there exist multiple solutions for* $\mathbf{C}$ *and* $\mathbf{D}$.

Next, we restrict attention to the empirical relevant parameter space $\alpha > 0$, $\beta > 0$, and $\beta^{-1} > \gamma \geq 0$. This allows us to relate the solutions to certain policy regimes, as we prove in Appendix D.

PROPOSITION 2. *For the empirically realistic case it holds that: in a PF/AMHE regime determinacy prevails. A PF/PMHE regime results in local indeterminacy or divergence, depending on the share of agents with RE. An AF/AMHE regime yields local divergence. Moreover, an AF/PMHE regime may lead to determinacy, if the share of agents with RE is sufficiently high. If this share is too low, the regime triggers local divergence.*

Intuitively local divergence occurs, because a policy regime fails to ensure that $0 < |K_1|, |K_2|, |K_3| < 1$ in (16). Consequently, one or more of the coefficients are larger than one in modulus and the dynamics of π_t become explosive.

In Figure 1 (and animated Figure 2) below we numerically illustrate our findings of Propositions 1 and 2 in the α-γ-χ-space.

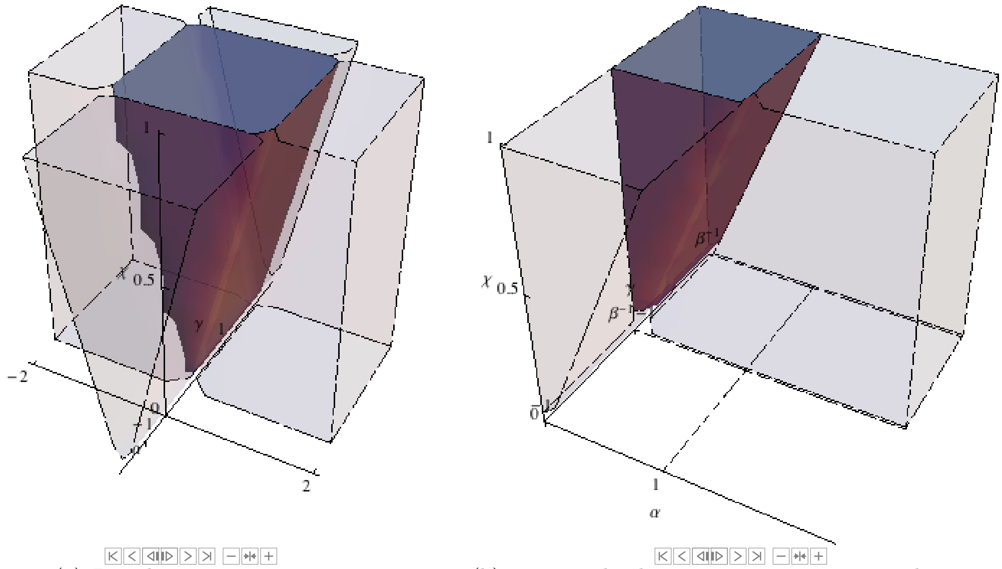


(a) Broad parameter space.

(b) Empirical relevant parameter space with $\alpha \geq 0$, $0 \leq \gamma < \beta^{-1}$.

Figure 1: Regions of local determinacy (light grey), indeterminacy (dark grey), and explosiveness(remainder) in the α-γ-χ-space for $\iota = 0.9$ and $\beta = 0.99$.

Its surface, corresponding to $\chi = 1$, represents an illustration of the results obtained by Evans and Honkapohja (2007) for the homogeneous RE benchmark case. The additional implications of heterogeneous expectations for the dynamics of the economy become evident, once we consider the cases of $\chi < 1$. In particular, the region of approximately $-1 < \alpha < 1$ and below $\chi \approx 0.5$. In this area of the parameter space the PF/PMHE regime, and more important, the AF/PMHE regime have fundamentally different dynamic properties as is known from homogeneous expectations benchmark, i.e. local explosiveness.



(a) Broad parameter space.

(b) Empirical relevant parameter space with $\alpha \geq 0$, $0 \leq \gamma < \beta^{-1}$.

Figure 2: Regions of local determinacy (light grey), indeterminacy (dark grey), and explosiveness(remainder) in the α-γ-χ-space animated for $\iota \in \{0.2, 0.3, ..., 1.8\}$ with starting value $\iota = 0.2$ and $\beta = 0.99$. (The animations may not be correctly displayed under all operating systems and PDF viewers. More information is available from the author upon request.)

Moreover, recall that we report animations for a wide range of ι in Figure 2. They indicate that, as long as agents with non-rational expectations have forecasts that are a function of past data, their share is decisive, not their particular functional form, e.g. ι larger or smaller than one.

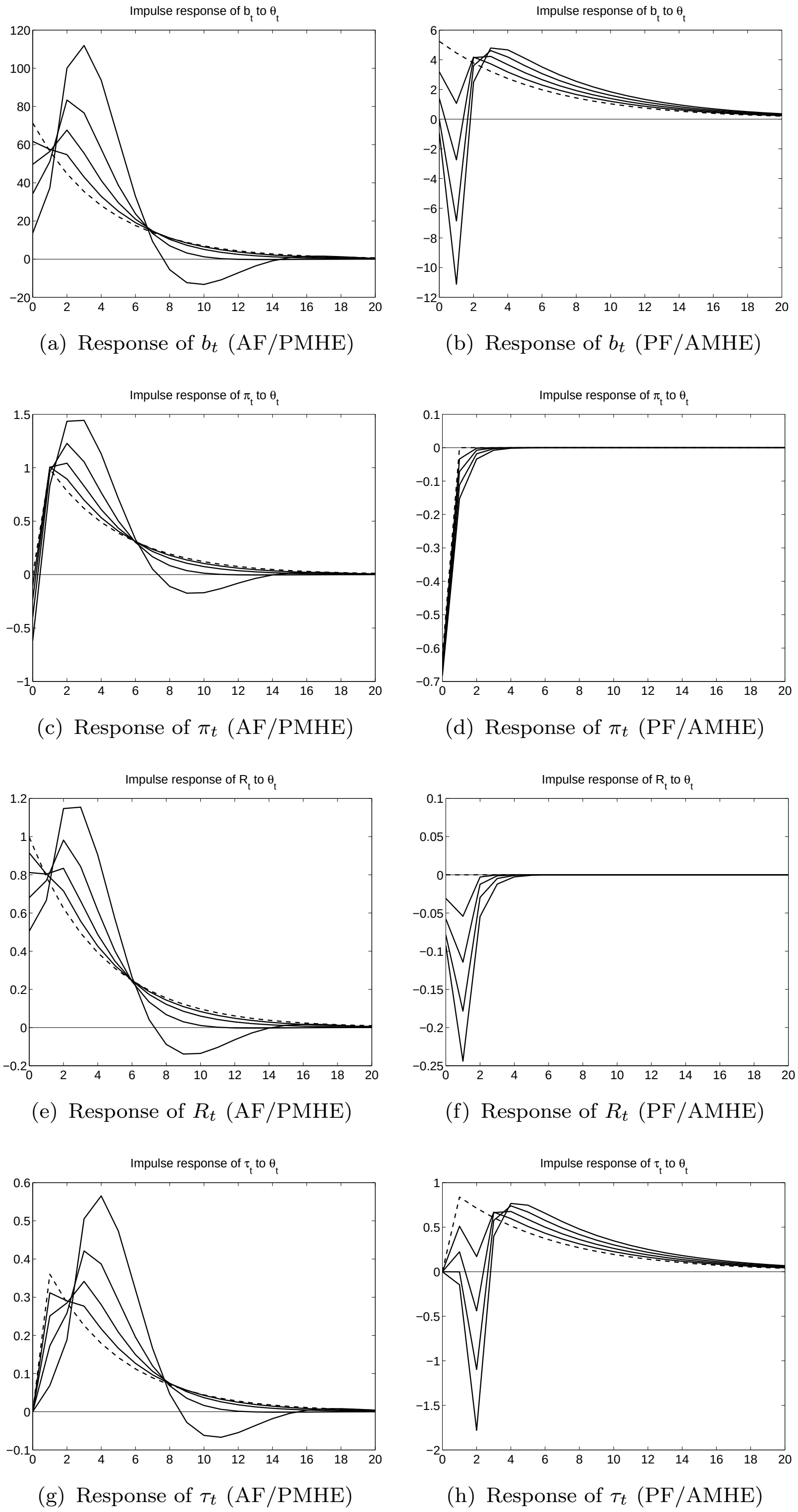
3.2. Simulated Impulse Responses to Transitory Policy Shocks

Policy interaction in this economy also involves responses of the policy instrument of one institution to an exogenous shock to the instrument of the other institution. Thus, it is quite natural to ask, how these exogenous policy shocks propagate through the economy under certain policy regimes, once heterogeneous expectations are present. We address this question by means of simulated impulse responses. For this analysis we use the calibration as outlined in Table 1.

Table 1. Calibration

Parameter	Value
α	$\in \{0.8, 1.6\}$
γ_0	0.50
γ	$\in \{(\beta^{-1} - 1) + 0.15, (\beta^{-1} - 1)/2\}$
$\mathcal{A}$	0.10
β	0.99
σ_1	0.95
σ_2	0.95
y	10.00
g	1.50
π	1.10

The calibration is the same as in Evans and Honkapohja (2005), execpt for α and β. The former is chosen to implement either AMHE or PMHE. The latter puts the simulation in a quarterly context. Moreover, the choices of γ yield an AF or PF stance. Steady-state values for R, m, b, and $\tilde{C}$ are calculated as outlined in the Evans and Honkapohja (2007, p.688).



(a) Response of b_t (AF/PMHE) (b) Response of b_t (PF/AMHE)

(c) Response of π_t (AF/PMHE) (d) Response of π_t (PF/AMHE)

(e) Response of R_t (AF/PMHE) (f) Response of R_t (PF/AMHE)

(g) Response of τ_t (AF/PMHE) (h) Response of τ_t (PF/AMHE)

Figure 3: Impulse responses to a contractionary one standard deviation transitory *monetary* policy shock, θ_t under both regimes. Expectations are calibrated to $\iota = 0.9$ with the solide lines representing the cases $\chi \in \{0.6, ..., 0.9\}$. The dashed line is for the RE benchmark case $\chi = 1$.

Figure 3 illustrates the simulated impulse responses of the endogenous variables to a contractionary one standard deviation transitory monetary policy shock, θ_t.[6] The panels in the left column are for the AF/PMHE regime, the ones in the right column are for the PF/AMHE regime.

For both regimes, the impact effects on π_t have larger magnitude for the case of heterogeneous expectations. Next, under heterogeneous expectations, impact effects can even have the opposite sign of the one under homogeneous RE. Moreover, the impulse responses are more persistent, when χ decreases. Independent of this fact, impulse responses for π_t are also more persistent under the AF/PMHE regime compared to the PF/AMHE regime. However, the most striking feature under the former regime are the dampening oscillations with increasing amplitude, which emerge for all variables with decreasing χ. In contrast, under the PF/AMHE one observes monotonic convergence for π_t and R_t and eventual undershooting of the steady-state for b_t and τ_t.

A careful inspection of (16) under the respective policy regime provides the intuition for the aforementioned observations. Notice that, given our calibration, $K_1 > 0$, $K_2 < 0$, and $K_3 > 0$ as well as $\partial K_1/\partial\chi < 0$, $\partial K_2/\partial\chi > 0$, and $\partial K_3/\partial\chi < 0$. Now, in case of the PM/AMHE regime restriction (*ii*) applies. Therefore the coefficient K_2 yields a decrease in π_t on impact for $\chi = 1$ and an even stronger decrease on impact, if $\chi < 1$, as one can observe in Panel 3(d). While $\chi = 1$ implies $K_3 = 0$, this coefficient increases, when χ decreases and in this way lagged inflation generates the persistence in the impulse responses. Thus, expectational heterogeneity ultimately causes persistent responses to a monetary policy shock.

[6] We restrict the analysis to transitory shocks on purpose. It allows us to illustrate the persistence generated by heterogeneous expectations.

Consequently, R_t does not respond in case of $\chi = 1$ as one can see from Panel 3(f). The reason is that feedback to inflation perfectly offsets the direct effect of the shock on the instrument. In case of $\chi < 1$, the initial drop in π_t is larger and the feedback to this drop outweighs the direct effect of θ_t on R_t. Likewise, π_t responds persistently and so does R_t, while giving feedback to the former.

The impact effects on π_t and R_t in turn explain the responses of b_t on impact in Panel 3(b). The more negative is the net effect of the former two variables, the smaller is the response of b_t on impact. While π_t returns to the steady-state, b_t decreases further. Together with the lagged response of τ_t to b_t, the responses of π_t and R_t also support the relatively slow return of b_t to its steady-state value. The overshooting of b_t is driven by the lagged response of τ_t to deviations in b_t.

For the AF/PMHE regime restriction (i) holds. Now consider the case of $\chi = 1$. Given the characteristics of the coefficients K_i, the monetary policy shock causes a drop in π_t on impact as Panel 3(c) illustrates. Note that its direct negative effect on π_t outweighs the positive effect of an increase in b_t on impact. Furthermore, Panel 3(e) exhibits the weak policy feedback to π_t due to PMHE. In consequence R_t increases on impact.

In the subsequent periods, (16) implies a persistent convergence of π_t from above, as there is a positive effect of current b_t. R_t follows this development. Moreover, τ_t in Panel 3(g) shows a lagged response to the increase b_t. However, it is smaller compared to the PF/AMHE regime, which is consistent with AF. This and the adjustment in π_t drive b_t back to the steady-state.

Now, in case of $\chi < 1$, Panel 3(c) illustrates an amplified impact effect on π_t. This is consistent with a more negative K_2 and a less positive K_1. The extent of amplification on impact also determines the extent to which the responses of R_t and b_t are mitigated on impact.

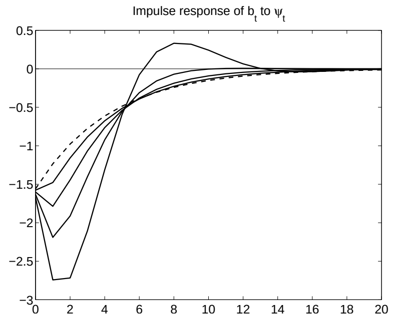
Following the impact effects, next to the positive effect of current b_t the presence of heterogeneous expectations also implies a negative effect of lagged π_t. If χ is small enough, e.g. $\chi = 0.6$, the latter effect creates dampening oscillations in π_t. These dynamics of π_t in turn lead the oscillations in b_t, R_t, and τ_t through exactly the same channels as discussed for the case of $\chi = 1$. Thus, the ultimate source of the oscillations is the expectational heterogeneity, as it triggers an interplay between b_t and π_{t-1} in the inflation dynamics.

Next, Figure 4 exhibits the simulated impulse responses to a negative one standard deviation transitory fiscal policy shock, ψ_t. The impact effects now are identical independent of χ. Furthermore, under the PF/AMHE regime, π_t and R_t are not affected by the fiscal policy shock, which must hold by construction of the monetarist solution. Only τ_t responds to a rise in b_t. The opposite must be, and is indeed true under the AF/PMHE regime, where π_t is affected by b_t. In this case, the variables converge monotonically towards the steady-state for $\chi = 1$, whereas $\chi < 1$ eventually causes dampening oscillations once more.

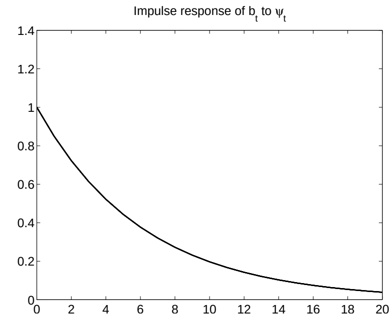
Again, examining (16) helps to clarify these observations. Clearly π_t cannot be affected by a fiscal policy shock in case of the PF/AMHE regime, as coefficient $K_1 = 0$ and the other terms are irrelevant. This applies for $\chi \leq 1$ and explains, why π_t, and consequently R_t, do not respond at all. As π_t does not respond to the shock, b_t rather mechanically rises due to the transitory drop in τ_t, as one can spot in Panels 4(b) and 4(h). In the subsequent periods, τ_t is raised, as PF feeds back sufficiently strong to offset the rise in b_t.

In case of the AF/PMHE regime the negative fiscal policy shock from Panel 4(g) causes b_t to decrease on impact, which also triggers a drop in π_t via (16) and a negative response of R_t on impact. Under $\chi = 1$ monotonic convergence towards the steady-state follows, as $K_3 = 0$. Moreover, τ_t shows (almost) no response to b_t below its steady-state due to the AF stance. In contrast, $\chi < 1$, and in consequence $K_3 > 0$, make π_t dependent on its lagged value during the transition. The latter once more creates reciprocity between b_t and π_{t-1}, which potentially once more generates dampening oscillations in π_t, which are then followed by b_t and R_t.

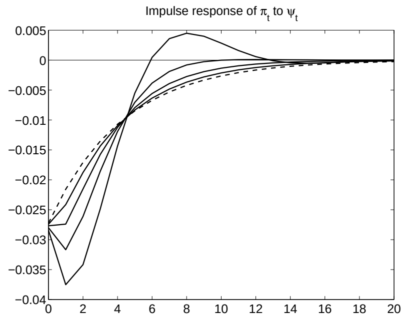
Why does b_t fall on impact? Because the negative tax shock creates a positive income effect at the beginning of the period. Thus, households will want to save less, and government can issue less bonds. This is reflected in a negative deviation of end-of-period b_t. Is this consistent with the impulse responses under the PF/AMHE regime? Indeed; in contrast to the AF stance, households anticipate that there will be above steady-state taxes in the future under PF. Thus, there is a negative wealth effect under the PF stance. Hence, the government can partially offset the temporary drop in τ_t by issuing more b_t, as households want to save more.



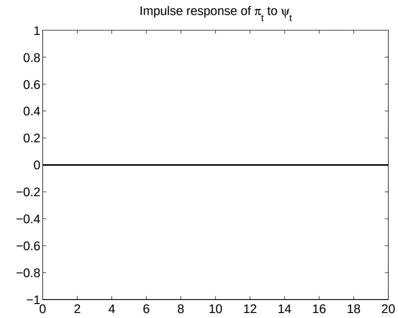
(a) Response of b_t (AF/PMHE)



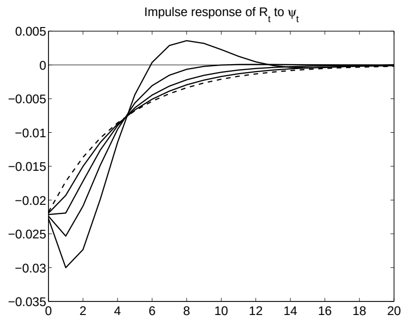
(b) Response of b_t (PF/AMHE)



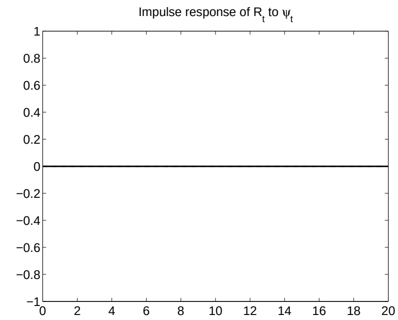
(c) Response of π_t (AF/PMHE)



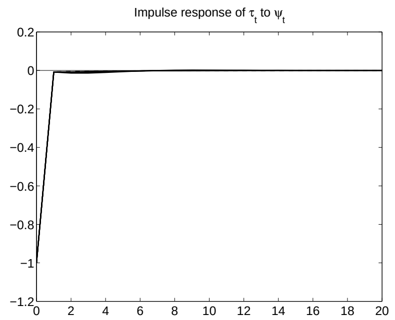
(d) Response of π_t (PF/AMHE)



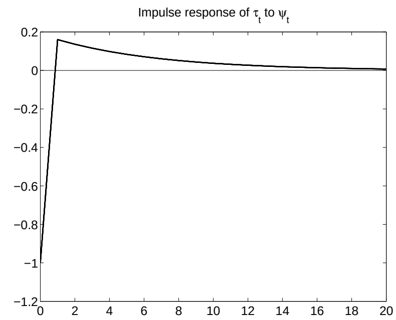
(e) Response of R_t (AF/PMHE)



(f) Response of R_t (PF/AMHE)



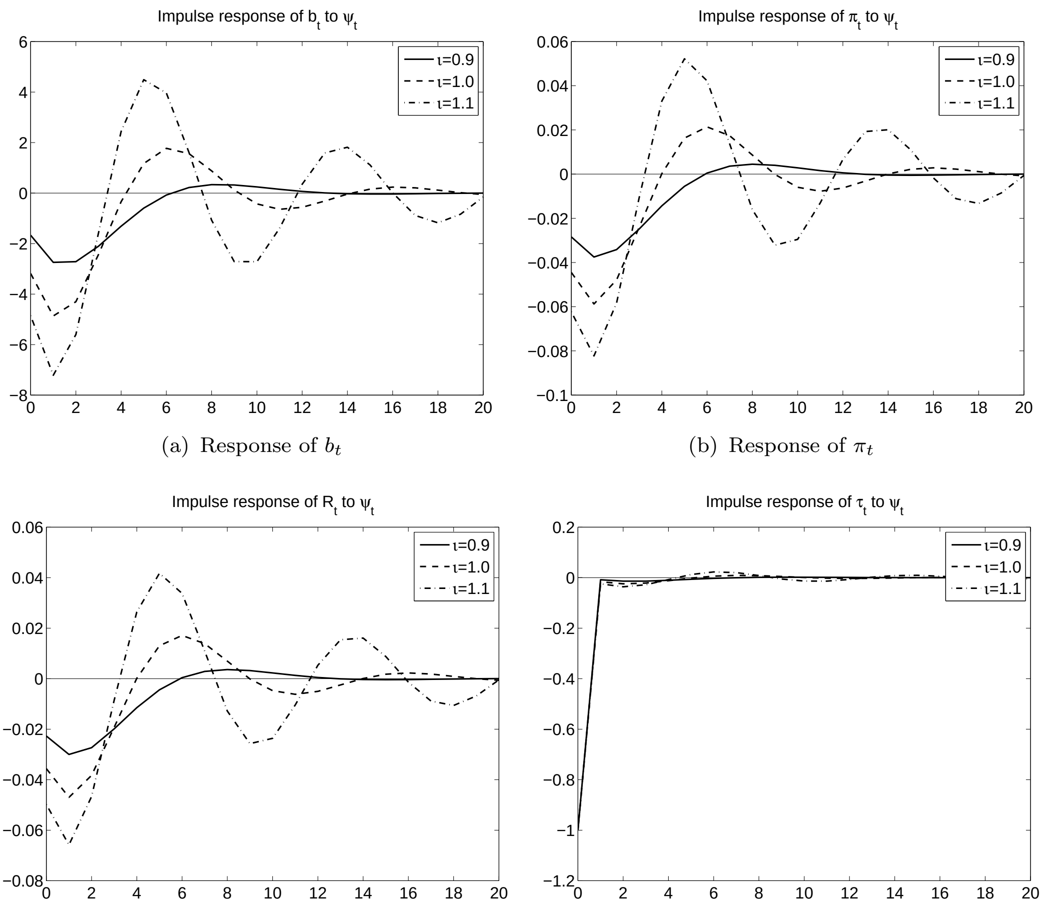
(g) Response of τ_t (AF/PMHE)



(h) Response of τ_t (PF/AMHE)

Figure 4: Impulse responses to a negative one standard deviation transitory *fiscal* policy shock, ψ_t under both regimes. Expectations are calibrated to $\iota = 0.9$ with the solide lines representing the cases $\chi \in \{0.6, ..., 0.9\}$. The dashed line is for the RE benchmark case $\chi = 1$.

Finally, notice that the emergence of the dampening oscillations under the AF/PMHE regime is only modestly dependent on whether the coefficient ι is below or above unity. We illustrate this in Figure 5 for the negative one standard deviation transitory fiscal policy shock. This is important, as the choice of ι usually turns out to discriminate between fundamentally different dynamics in homogeneous expectations economies. In our case, values of ι above or below one affect the amplitude and frequency of the dampening oscillations. However they are not decisive for whether or not the oscillations occur.



(a) Response of b_t

(b) Response of π_t

(c) Response of R_t

(d) Response of τ_t

Figure 5: Impulse responses to a negative one standard deviation transitory *fiscal* policy shock, ψ_t under an AF/PMHE regime for $\chi = 0.6$ and $\iota \in \{0.9, 1.0, 1.1\}$.

4. DISCUSSION OF RESULTS

Our findings have various economic implications. First and foremost, the PF/AMHE regime in principal yields determinacy. However, heterogeneous expectations impose an informational challenge on the central bank. It needs to respond sufficiently strong to inflation, which entails to successfully track private sector expectations. This requirement can eventually be met by modern central banks. In fact, central banks make a great effort to track private sector expectations. Also notice that for the homogeneous RE benchmark case, AMHE means nothing but $\alpha > 1$, which is equivalent to AM. Thus, one may regard our result as an extension of the Taylor (1993)-principle in a heterogeneous expectations set-up.

Second, the PF/PMHE, in theory, may be a more dangerous regime than is known under homogeneous RE. In this case both the fiscalist and the monetarist solution are stationary for the benchmark case $\chi = 1$. However, when the share of agents with AE becomes sufficiently high, this regime leads to divergence. Note that this finding is broadly in line with the one of Evans and Honkapohja (2007, p.681) that neither solution is stable under homogeneous recursive least squares learning. However, in our case π_t and b_t become complex, and therefore the possibility of explosiveness under this regime can be considered unrealistic.

Third, our analysis confirms the finding of the homogeneous expectations literature on fiscal and monetary policy interaction that an AF/AMHE regime leads to local explosiveness.

Fourth, considering the latter result and likewise acknowledging the fact that it is usually the central bank that is more flexible and faster in implementing policy changes, our result for the AF/PMHE regime deserves special attention. Based on the homogeneous RE benchmark case, on may argue that, once fiscal policy switches from PF to AF, the central bank can bring about determinacy by switching from AM to PM. However, an AF/PMHE regime makes the economy prone to local divergence, if roughly the majority of agents has AE. This is in the range of the share of agents with AE documented in survey data by Branch (2004).

It is important to emphasize that our results for the AF/PMHE regime do not question the plausibility of the fiscalist solution, but demonstrate its vulnerability to the existence of heterogeneous expectations. Thus, one can also view our findings as a challenge to conventional fiscal rules like (10) and as an argument in favour of fiscal rules that account for private sector expectations. The latter may eventually safeguard the economy against explosive dynamics in inflation.

Fifth, the occurrence of persistent impulse responses, in our Neo-Classical model, especially to monetary policy shocks, is worthwhile. It states an example of how persistence can be a feature of an economic model, which does not rely on nominal rigidities. This is of particular interest in light of the debate on the plausibility of nominal rigidities, see e.g. De Grauwe (2010a,b, 2012) and others.

Finally, the dampening oscillations in the simulated impulse responses to fiscal and monetary policy shocks under the AF/PMHE regime, indicate that heterogeneous expectations might be one potential source to explain business cycles.

Arguably, judging the generality of the results regarding the simulated impulse response analysis requires a critical acclaim of our key assumption, i.e. the nature of heterogeneity. We consider constant shares of agents with RE or AE. As emphasized in the introduction, our motivation is empirical evidence, analytical tractability and comparability.

At the same place we highlight that this set-up can be viewed as one of persistent heterogeneity. Evans and Honkapohja (2013) argue that this may be a plausible assumption, even when individuals deliberate various forecasting models and may freely switch between them. At the heart of the argument is the key assumption of the dynamic predictor selection literature pioneered by Brock and Hommes (1997). Each forecasting model is not only evaluated based on how well it fits the date, but is also associated with some information and/or usage cost. Exactly this trade-off justifies to assume persistent heterogeneity.

However, modeling this discrete choice in our framework, would imply that χ may vary over time. According to our view, the main implication of a time varying χ would not only be that the determinacy properties may change fundamentally over time. It also could imply that, conditional on determinacy, the impulse responses may exhibit time-varying characteristics with regard to persistence and oscillations.

5. CONCLUSIONS

In sum, this paper puts forth a Neo-Classical theory of fiscal and monetary policy interaction under heterogeneous expectations. The coexistence of agents with RE and AE gives rise to economic dynamics strikingly different from the homogeneous RE benchmark case.

For plausible assumptions on the parameter space, we show that the monetarist solution can be the unique stationary RE solution in a PF/AMHE regime. This is true, as the central bank obeys a generalized Taylor (1993)-principle by incorporating knowledge about the heterogeneous nature of private sector expectations.

Also the fiscalist solution, where inflation depends on public debt, can be the unique stationary RE solution, given there is an AF/PMHE regime in place. Nevertheless, under this regime, ultimately the shares of agents with RE and AE become decisive for stationarity, as fiscal policy does not account for expectational heterogeneity. If the share of agents with RE goes below the empirically relevant range of one half, the fiscalist solution becomes explosive. This stands in sharp contrast to our findings for the monetarist solution. The result suggests that the AF/PMHE regime is not a very desirable one, if it is based on an arguably conventional fiscal rule. Thus, the result calls for a modified fiscal policy approach, which incorporates private sector expectations.

Moreover, we find that an AF/AMHE regime leads to local divergence and a PF/PMHE regime opens the door to arbitrary large economic fluctuations associated with indeterminacy.

Finally, we present simulated impulse responses to transitory fiscal and monetary policy shocks. Our computations indicate that persistent responses can be a feature of a Neo-Classical economy. In case of the fiscalist solution, dampening oscillations in inflation and other endogenous variables emerge. Both characteristics of the responses are solely driven by the coexistence of two different types of expectations.

We believe that the concern of persistent expectational heterogeneity and bounded rationality in general, and with regard to policy interaction in particular, is of high relevance for academics as well as policy makers. One can view the present paper as a very general way of addressing this concern. Clearly, our modeling approach is highly stylized and might neglect important aspects. One exemplary issue might be that economic agents might have a discrete choice among various forecasting models. Then, following the approach of Brock and Hommes (1997), one can imagine that agents switch their forecasting function from time to time. As the shares of agents with RE and AE in the economy can become decisive for stationarity under a policy regime, one may expect even more forceful business cycle dynamics from such a modeling approach. Thus, we suppose that the variation of shares of different types of forecasters over time and its implications for policy interaction is an important topic, and calls for further investigation.

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A. THE SUBJECTIVE EXPECTATIONS OPERATOR

Note that the assumptions that are taken on expectations are word by word the assumptions A1 to A7 in Branch and McGough (2009, p.1038). We restate them for the convenience of the interested reader.

A1. Expectations operators fix observables.

A2. If x is a variable forecasted by agents and has steady-state $\bar{x}$ then $E^1\{\bar{x}\} = E^2\{\bar{x}\} = \bar{x}$.

A3. If x, y, $x+y$ and χx are variables forecasted by agents then $E_t^\gamma\{(x+y)\} = E_t^\gamma\{x\} + E_t^\gamma\{y\}$ and $E_t^\gamma\{\chi x\} = \chi E_t^\gamma\{x\}$.

A4. If for all $k \geq 0$, x_{t+k} and $\sum_{k=0}^{\infty} \beta^{t+k} x_{t+k}$ are forecasted by agents, then

$$E_t^\gamma \left\{ \sum_{k=0}^{\infty} \beta^{t+k} x_{t+k} \right\} = \sum_{k=0}^{\infty} \beta^{t+k} E_t^\gamma \{x_{t+k}\}.$$

A5. E_t^γ satisfies the law of iterated expectations: If x is a variable forecasted by agents at time t and time $t+k$ then $E_t^\gamma \circ E_{t+k}^\gamma\{x\} = E_t^\gamma\{x\}$.

A6. If x is a variable forecasted by agents at time t and time $t+k$ then $E_t^{\gamma} E_{t+k}^{\gamma'}\{x_{t+k}\} = E_t^{\gamma}\{x_{t+k}\}, \gamma \neq \gamma'$.

A7. All agents have common expectations on expected differences in limiting wealth.

B. MODEL DERIVATIONS

Consider the household's problem. We define $W_{t+1} \equiv m_t + b_t$ and $x_{t+1} = m_t$. Then the household's problem can be solved by the very same Lagrangian as in Evans and Honkapohja (2007), i.e.

$$\mathcal{L} = E_t^{\gamma} \left\{ \sum_{t=0}^{\infty} \beta^t \left[(1-\sigma_1)^{-1} c_t^{(1-\sigma_1)} + \mathcal{A}(1-\sigma_2)^{-1} (x_t \pi_t^{-1})^{(1-\sigma_2)} \right] \right.$$
$$-\beta^{t+1} \mu_{1,t+1} \left[W_{t+1} - y + c_t + \tau_t - x_t \pi_t^{-1} - R_{t-1} \pi_t^{-1} (W_t - x_t) \right]$$
$$\left. -\beta^{t+1} \mu_{2,t+1} \left[x_{t+1} - m_t \right] \right\}. \tag{B.1}$$

This yields the first-order conditions

$$E_t^{\gamma}\{c_t^{-\sigma_1}\} - \beta E_t^{\gamma}\{\mu_{1,t+1}\} = 0, \tag{B.2}$$

$$E_t^{\gamma}\{\mu_{2,t+1}\} = 0, \tag{B.3}$$

$$\beta^{-1} R_{t-1}^{-1} E_t^{\gamma}\{\mu_{1,t}\} = E_t^{\gamma}\{\mu_{1,t+1} \pi_t^{-1}\}, \tag{B.4}$$

$$E_t^{\gamma}\{\mu_{2,t}\} = \mathcal{A} E_t^{\gamma}\{\pi_t^{-1} (x_t \pi_t^{-1})^{-\sigma_2}\} + \beta E_t^{\gamma}\{(\pi_t^{-1} - R_{t-1} \pi_t^{-1}) \mu_{1,t+1}\}, \tag{B.5}$$

where we make use of *Assumption A3.* Re-arranging terms within (B.5), plugging in (B.4), forwarding the resulting expression and combining it with (B.2)-(B.3) yields

$$E_t^\gamma\{\mu_{2,t}\} = \mathcal{A}E_t^\gamma\{\pi_t^{-1}(x_t\pi_t^{-1})^{-\sigma_2}\} + \beta E_t^\gamma\{(\pi_t^{-1} - R_{t-1}\pi_t^{-1})\mu_{1,t+1}\}. \qquad \text{(B.6)}$$

If every agent can observe his own period t choice of c_t, and within-type expectations are identical, then in fact $E_t^\gamma\{c_t^{-\sigma_1}\} = c_t^{-\sigma_1}$, and we can use (B.2) and (B.4) to derive

$$c_t^{-\sigma_1} = \beta R_t E_t^\gamma\{c_{t+1}^{-\sigma_1}\pi_{t+1}^{-1}\}, \qquad \text{(B.7)}$$

where R_t is set by the central bank and states publicly available information. Goods market clearing implies that $c_t = y - g$, thus, by *Assumption A1*

$$c_t^{-\sigma_1} = (y-g)^{-\sigma_1}\beta R_t E_t^\gamma\{\pi_{t+1}^{-1}\}, \qquad \text{(B.8)}$$

and a within-type Fisher relation follows:

$$R_t^{-1} = \beta E_t^\gamma\{\pi_{t+1}^{-1}\}. \qquad \text{(B.9)}$$

Finally, money market equilibrium implies a within-type identity

$$\mathcal{A}\beta E_t^\gamma\{\pi_{t+1}^{\sigma_2-1}m_t^{-\sigma_2}\} = (y-g)^{-\sigma_1}(1 - \beta E_t^\gamma\{\pi_{t+1}^{-1}\}). \qquad \text{(B.10)}$$

C. DETERMINACY CONDITIONS AND LINEAR RESTRICTIONS

System (14) can be rewritten as

$$\begin{bmatrix} x_t \\ z_t \\ x_{t-1} \end{bmatrix} = \mathbf{\Lambda} \begin{bmatrix} x_{t+1} \\ z_{t+1} \\ x_t \end{bmatrix} + \mathbf{Q}^{-1} \left[\mathbf{F}_1 \eta_{t+1} + \mathbf{F}_2 \theta_{t+1} + \mathbf{F}_3 \theta_t + \mathbf{F}_4 \psi_{t+1}\right], \qquad \text{(C.1)}$$

where (C.1) follows from diagonalizing matrix $\mathbf{J}$ in (14). Note that $E_t \tilde{\pi}_{t+1} = \tilde{\pi}_{t+1} - \eta_{t+1}$, $\mathbf{J} = (\mathbf{Q\Lambda Q}^{-1})$ is a decomposition of $\mathbf{J}$ into its eigenvalues and its right eigenvector, and $[x_{t+1}\ z_{t+1}\ x_t]' = \mathbf{Q}^{-1}[\tilde{\pi}_{t+1}\ \tilde{b}_{t+1}\ \tilde{\pi}_t]'$.

The important matrices in (C.1) are given by

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}, \quad \text{and} \qquad \text{(C.2)}$$

$$\mathbf{Q}^{-1} = \begin{pmatrix} \frac{\beta(\beta\gamma-1)\chi(\alpha\beta\varphi_1+\varphi_2)}{(\alpha(\beta\gamma-1)+\Theta)\beta^2+(\beta\gamma-1)^2\chi} & 1 & \frac{\beta^2\Theta(\alpha\beta\varphi_1+\varphi_2)}{(\alpha(\beta\gamma-1)+\Theta)\beta^2+(\beta\gamma-1)^2\chi} \\ \frac{\chi}{\sqrt{\alpha^2\beta^2-4\Theta\chi}} & 0 & -\frac{\Theta}{\sqrt{\alpha^2\beta^2-4\Theta\chi}}\lambda_2 \\ -\frac{\chi}{\sqrt{\alpha^2\beta^2-4\Theta\chi}} & 0 & \frac{\Theta}{\sqrt{\alpha^2\beta^2-4\Theta\chi}}\lambda_3 \end{pmatrix}, \qquad \text{(C.3)}$$

where $\Theta \equiv (1-\chi)\iota^2$. $\lambda_1 \equiv (\beta^{-1}-\gamma)^{-1}$, $\lambda_2 \equiv \frac{(\alpha\beta)-\sqrt{(\alpha\beta)^2-4\Theta\chi}}{2\Theta}$, and $\lambda_3 \equiv \frac{(\alpha\beta)+\sqrt{(\alpha\beta)^2-4\Theta\chi}}{2\Theta}$ are the eigenvalues of $\mathbf{J}$.

Paralleling the analysis of Evans and Honkapohja (2007), from (C.1), and given $[C_1, C_2, C_3]' = -\mathbf{Q}^{-1}\mathbf{F}_3$ we can figure out three different cases. First, given an AF regime, $|(\beta^{-1}-\gamma)^{-1}| < 1$, stationarity of the solution requires that $E_t x_{t+1} = \lambda_1^{-1}(x_t + C_1\theta_t) = 0$ to rule out that $|E_t x_{t+s}| \to \infty$ as $s \to \infty$. This yields restriction (i). Moreover, in the PF/AMHE regime, where $|(\alpha\beta)| > \chi + \Theta$

is true, stationarity of the solution requires that $E_t z_{t+1} = \lambda_2^{-1}(z_t + C_2\theta_t) = 0$ to rule out that $|E_t z_{t+s}| \to \infty$ as $s \to \infty$. Restriction (ii) follows. Finally, in the PF/PMHE regime, where $|(\alpha\beta)| < \chi + \Theta$ is true, stationarity of the solution requires that $x_t = \lambda_3^{-1}(x_{t-1} + C_3\theta_t) = 0$ to rule out that $|x_{t+s}| \to \infty$ as $s \to \infty$. This leads to restriction (iii).

D. PROOF OF PROPOSITION 2

Proof. We consider the empirical relevant parameter space to be $\alpha > 0$, $\beta > 0$, and $\chi \in (0, 1]$. Moreover, following the arguments in Evans and Honkapohja (2007, p.681), we assume $\beta^{-1} > \gamma \geq 0$. The characteristic polynomial of $\mathbf{J}$ is given by

$$\begin{aligned}\mathcal{P}(\psi) = &-\psi^3 + \left[(\beta^{-1} - \gamma)^{-1} + \Theta^{-1}(\alpha\beta)\right]\psi^2 \\ &- \left[\Theta^{-1}((\alpha\beta)(\beta^{-1} - \gamma)^{-1} + \chi)\right]\psi + \Theta^{-1}\chi(\beta^{-1} - \gamma)^{-1},\end{aligned} \tag{D.1}$$

where it's roots coincide with the eigenvalues λ_1, λ_2, and λ_3. The assumptions on γ above imply that there is at least one real root, λ_1.

Moreover, *Descartes' rule of signs* suggests that there is a maximum of three positive real roots and zero negative real roots. Furthermore note that $\mathcal{P}(-\infty) \to +\infty$, $\mathcal{P}(-1) > 0$, $\mathcal{P}(0) > 0$, and $\mathcal{P}(\infty) \to -\infty$.

Next, with regard to λ_2 and λ_3, if $(\alpha\beta) > \chi + \Theta$, then $\mathcal{P}(1) < 0$, and either there is one real root or a pair of complex conjugates with the same modulus inside the unit circle. In case of $(\alpha\beta) < \chi + \Theta$, then $\mathcal{P}(1) > 0$, and there is no real root inside the unit circle. However, λ_2 and λ_3 may also form a pair of complex conjugates. In this case their identical modulus can be inside or outside the unit circle. In order to analyze the various possible cases, it is useful to calculate the discriminant of $\mathcal{P}(\psi)$, which is given by

$$\mathcal{D} = \frac{(\alpha^2\beta^2 - 4\Theta\chi)\left[\beta^2(\alpha(\beta\gamma-1)+\Theta)+\chi(\beta\gamma-1)^2\right]^2}{\Theta^4(\beta\gamma-1)^4}. \tag{D.2}$$

According to Irving (2004, p.154), three cases are possible. First, if $\mathcal{D} > 0$, then $\mathcal{P}(\psi)$ has three distinct real roots. Second, if $\mathcal{D} < 0$, then $\mathcal{P}(\psi)$ has one real root and a pair of complex conjugates with identical modulus. We ignore the third case, where $\mathcal{D} = 0$ and $\mathcal{P}(\psi)$ has multiple real roots. One can verify that the sign of $\mathcal{D}$ depends on whether $(\alpha\beta)$ is larger or smaller than $\sqrt{4\chi\Theta}$. Furthermore, note that $\chi + \Theta \geq \sqrt{4\chi\Theta}$.

Now, in case of PF, i.e. $\gamma > \beta^{-1} - 1$, the root λ_1 is real and outside the unit circle. Likewise root λ_1 is real and inside the unit circle in case of AF, i.e. $\gamma < \beta^{-1} - 1$.

Consequently, in a PF/AMHE regime it follows that $(\alpha\beta) > \chi + \Theta \geq \sqrt{4\chi\Theta}$ and there are three distinct real roots, $|\lambda_1| > 1$, $|\lambda_2| < 1$, and $|\lambda_3| > 1$ which yield local determinacy.

In contrast, under an AF/AMHE regime there is local divergence from the steady-state as this policy regime yields $|\lambda_1| < 1$, $|\lambda_2| < 1$, and $|\lambda_3| > 1$.

Next, given a PF/PMHE regime, it is true that, when $\chi + \Theta > (\alpha\beta) > \sqrt{4\chi\Theta}$, there are three distinct real roots, $|\lambda_1| > 1$, $|\lambda_2| > 1$, and $|\lambda_3| > 1$ and this results in local indeterminacy. In case of $\chi + \Theta \geq \sqrt{4\chi\Theta} > (\alpha\beta)$ there is a pair of complex conjugates, λ_2 and λ_3, with identical modulus. If $\lambda_2\lambda_3 = \chi/\Theta < 1$, then their identical modulus is inside the unit circle. If $\lambda_2\lambda_3 = \chi/\Theta > 1$, then it is outside the unit circle.

In sum, when $\chi + \Theta \geq \sqrt{4\chi\Theta} > (\alpha\beta)$ is true, a PF/PMHE regime leads to local indeterminacy if $\chi/\Theta > 1$, as $|\lambda_1|, |\lambda_2|, |\lambda_3| > 1$. And, if $\chi/\Theta < 1$ there is local divergence from the steady-state as $|\lambda_1| > 1$, and $|\lambda_2|, |\lambda_3| < 1$.

Finally, for the AF/PMHE regime similar arguments apply. In case of $\chi + \Theta > (\alpha\beta) > \sqrt{4\chi\Theta}$, there are three distinct real roots, $|\lambda_1| < 1$, and $|\lambda_2|, |\lambda_3| > 1$ and local determinacy prevails. However, when $\chi + \Theta \geq \sqrt{4\chi\Theta} > (\alpha\beta)$ is true, an AF/PMHE regime does only yield local determinacy if $\lambda_2\lambda_3 = \chi/\Theta > 1$, but results in local divergence if $\lambda_2\lambda_3 = \chi/\Theta < 1$

□